

Bosonic M Theory

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Abstract

We conjecture that there exists a strong coupling limit of bosonic string theory which is related to the 26 dimensional theory in the same way that 11 dimensional M theory is related to superstring theory. More precisely, we believe that bosonic string theory is the compactification on a line interval of a 27 dimensional theory whose low energy limit contains gravity and a three-form potential. The line interval becomes infinite in the strong coupling limit, and this may provide a stable ground state of the theory. We discuss some of the consequences of this conjecture.

1 Introduction

There is growing evidence that all the known perturbative ten dimensional superstring theories are limits of an eleven dimensional theory called M theory [1, 2, 3]. In particular, the ubiquitous dilaton which controls the string coupling is simply related to the size of the extra dimension. The 26 dimensional bosonic string has not been included in these developments mostly due to the widespread belief that the existence of the tachyon indicates that the theory is ill defined¹. However during the past year the significance of the open string tachyon has been understood. Rather than indicating that the theory is sick, it just shows that the usual vacuum is unstable. As Sen first proposed [6], this vacuum can be viewed as a closed string vacuum together with an unstable D 25-brane. There is increasing evidence that there is a stable minimum of the open string tachyon potential at a value equal to minus the tension of a D 25-brane, and about this minimum there are no open string excitations. This raises the possibility that the closed string tachyon can similarly be removed by appropriately shifting to a new ground state. However, there are good arguments that the closed string tachyon cannot be removed by direct analogy to the open string case. There is probably no stable minimum of the closed string tachyon potential [7]. Something more dramatic is needed.

In this paper we study the strong coupling limit of bosonic string theory and argue that the tachyon instability may be removed in this limit. Unfortunately we have very little firm ground to stand on when trying to determine the strong coupling limit of a theory without supersymmetry. In this paper we make a guess based on the assumption that bosonic string theory is not wholly dissimilar to IIA and heterotic string theory.

The main clue motivating our guess comes from the existence of the dilaton and its connection to the coupling constant. The action for the massless sector of bosonic string theory is

$$S = \int d^{26}x \sqrt{-g} e^{-2\phi} \left[R + 4 \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right] \quad (1.1)$$

Evidently, as in IIA string theory, the dilaton enters the action just as it would if it represented the compactification scale of a Kaluza Klein theory. We propose to take this seriously and try to interpret bosonic string theory as a compactification of a 27 dimensional theory. We will refer to this theory as bosonic M theory. (The possibility that the bosonic string has a 27 dimensional origin was also briefly discussed in [8], in the

¹The strong coupling limits of the ten dimensional bosonic Type 0 theories have been discussed in [4], and other nonsupersymmetric theories have been considered in [5].

context of a proposed matrix string formulation.)

In the case of IIA string theory there was a second clue that led to its interpretation as a Kaluza-Klein compactification of an 11 dimensional theory; the existence of a vector boson in the string spectrum. Closed bosonic string theory does not have a massless vector. This means it cannot be a compactification on an S^1 . In this respect the situation is more analogous to that of heterotic string theory. The solution to the strong coupling problem in that case is a compactification of M theory on a line interval, or more exactly on the orbifold S^1/Z_2 [9]. The absence of a $U(1)$ symmetry means that there is no massless gauge boson. Accordingly, we propose that closed bosonic string theory is a compactification of 27 dimensional bosonic M theory on S^1/Z_2 . In the supersymmetric case, to cancel anomalies one had to add E_8 gauge fields at each end of the line interval. In the bosonic case, since there are no fermions or chiral bosons, there are no anomalies to cancel. So there are no extra degrees of freedom living at the fixed points.

As in the case of the M theory - heterotic connection, the weakly coupled string theory is the limit in which the compactification length scale becomes much smaller than the 27 dimensional Planck length and the strong coupling limit is the decompactification limit. The 27 dimensional theory should contain membranes but no strings, and would not have a dilaton or variable coupling strength. The usual bosonic string corresponds to a membrane stretched across the compactification interval.

Some support for bosonic M theory comes from the following simple observation. The left moving modes of the heterotic string are precisely those of the 26 dimensional bosonic string. It has been argued [10] that at least perturbatively, the right moving modes of the bosonic string can be embedded in the right moving modes of the heterotic string. So the entire bosonic string is contained in the heterotic string. Since we now know that nonperturbatively the heterotic string grows an extra dimension, it is plausible that the bosonic string will similarly gain an extra dimension at strong coupling.

2 The Low Energy Theory

In this section we study the low energy limit of bosonic M theory which is a gravity theory in 27 dimensions. Without the powerful tool of supersymmetry it is difficult to give rigorous arguments. Nevertheless there are some plausible guesses that we can make about the form of the low energy action, using the fact that it must reduce to the usual bosonic string theory in the weak coupling limit. After deriving the low energy action, we

show how the tachyon instability may be removed in the strong coupling limit, and then study branes in this theory.

2.1 Motivation from Weak Coupling Limit

In order to reproduce the known spectrum of weakly coupled bosonic string theory, bosonic M theory will have to contain an additional field besides the 27 dimensional gravitational field, namely a three-form potential $C_{\mu\nu\rho}$. Let us consider the various massless fields that would survive in the weak coupling limit. First of all, there would be the 26 dimensional graviton. As usual, general covariance in 26 dimensions would insure that it remains massless. The component of the 27 dimensional gravitational field $g_{27,27}$ is a scalar in the 26 dimensional theory. It is of course the dilaton. No symmetry protects the mass of the dilaton. In fact we know that at the one loop level a dilaton potential is generated that lifts the dilatonic flat direction. Why the mass vanishes in the weak coupling limit is not clear.

Massless vectors have no reason to exist since there is no translation symmetry of the compactification space. This is obvious if we think of this space as a line interval. If we think of it as S^1/Z_2 then the two fixed points of the orbifold break the symmetry.

The three-form gauge field $C_{\mu\nu\sigma}$ gives some massless fields. If one of the indices of the three-form is in the compact 27^{th} direction, the resulting 26 dimensional field is the two-form $B_{\mu\nu}$ which is well known in bosonic string theory. It remains massless due to its gauge invariance. The components of $C_{\mu\nu\sigma}$ in which all three components are in the 26 dimensional subspace give a three-form which is absent in the usual bosonic string spectrum. Once again we take a hint from heterotic string theory. In that case the three-form that would be inherited from the 11 dimensional origin of heterotic string theory is projected away by the Z_2 identification. This is because M theory includes a Chern-Simons term which implies that the action is invariant under Z_2 only if C is odd under this identification. In the present case we will also assume that C is odd under the Z_2 .² Given our limited knowledge of the theory, we do not know if this is required by a symmetry of the action or not.

²If y is the coordinate along the S^1 , the fact that the basis vector $\partial/\partial y$ points away from the fixed point $y = 0$, means that it also must change sign under the Z_2 . This means that the components C_{yij} are even under $y \rightarrow -y$, while C_{ijk} are odd (where i, j, k denote all directions other than y).

We are thus led to the following low energy action for bosonic M theory:

$$S = \int d^{27}x \sqrt{-\hat{g}} \left[R(\hat{g}) - \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} \right] \quad (2.1)$$

where $F = dC$. To see the relation to (1.1), we set

$$\widehat{ds}^2 = e^{2\sigma} dy^2 + e^{-\sigma} g_{\mu\nu} dx^\mu dx^\nu \quad (2.2)$$

where σ and $g_{\mu\nu}$ are functions of x^μ but independent of y , and set

$$H_{\mu\nu\rho} = F_{y\mu\nu\rho} \quad (2.3)$$

The coordinate y takes values $-1 \leq y \leq 1$ and we identify y with $-y$. This prevents a term like $A_\mu dy dx^\mu$ from appearing in (2.2). Substituting into the action and integrating by parts yields

$$S = \int d^{26}x \sqrt{-g} e^{-11\sigma} \left[R(g) + 125 \nabla_\mu \sigma \nabla^\mu \sigma - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right] \quad (2.4)$$

There is no four-form in 26 dimensions since, as we have just explained, it is projected out by the identification on y . If we now define $2\phi \equiv 11\sigma$, this becomes the standard action for bosonic string theory (1.1) except that the coefficient of the $(\nabla\phi)^2$ is off by a factor of 125/121. So we recover the right fields and interactions, but one numerical coefficient is slightly off. This is not a contradiction since the action (2.1) is only valid on scales larger than the 27 dimensional Planck length, and to recover (1.1) we need to take a limit where one direction becomes much smaller than this. Without supersymmetry to protect coefficients, they can change as the coupling increases. In this respect, the factor of 125/121 may be analogous to the factor of 3/4 which arises in comparing the entropy of weakly coupled 3 + 1 Yang-Mills with the near extremal three-brane [11].

The relation between the 27 dimensional Planck length l_p and the 26 dimensional string length l_s and coupling $g = e^\phi$, follows from the relation between σ and ϕ . Since $g^2 l_s^{24} = G_{26} = G_{27}/e^\sigma l_p$ we get

$$g^{1/11} l_s = l_p \quad (2.5)$$

Since $g = e^{11\sigma/2}$, weak coupling corresponds to a small distance in the extra dimension, as expected.

There is a possibility of adding a cosmological constant to the action (2.1). Indeed, in the absence of supersymmetry, it would appear inevitable that one is generated. We will discuss this in section 4, but for now, we will assume the cosmological constant is zero.

2.2 Tachyon

We now consider the fate of the closed string tachyon at strong coupling. The trivial solution to (2.1) consisting of $F = 0$ and flat spacetime compactified on S^1/Z_2 has a non-perturbative instability. This is analogous to the instability of the Kaluza-Klein vacuum found by Witten [12], and very similar to its application to heterotic – M theory in [13]. The process that destabilizes the space is mediated by an instanton in which the two ends of the world (ends of the compactification interval) come together and produce a “hole” in space. In Minkowski space the hole rapidly grows and eats the entire space.

The appropriate instanton is (a projection of) the 27 dimensional euclidean Schwarzschild metric.

$$ds^2 = \left[1 - \left(\frac{r_0}{r}\right)^{24}\right] dy^2 + \left[1 - \left(\frac{r_0}{r}\right)^{24}\right]^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\Omega_{24}) \quad (2.6)$$

The coordinate y is periodic with period $P = \pi r_0/6$. To apply it to our case, we identify y and $-y$. So the size of the extra dimension at infinity is $\pi r_0/12$. To obtain the Lorentzian evolution, one analytically continues $\theta \rightarrow (\pi/2) + it$. To picture this evolution, consider the surfaces at the ends of the interval, $y = 0, \pi r_0/12$. The separation between these surfaces goes to zero smoothly at $r = r_0$, so the two surfaces are really one surface with the shape of a wormhole. At the initial time, $t = 0$, the proper size of the wormhole is r_0 , but as time evolves, it grows exponentially.

This instanton description is only valid for $r_0 \gg l_p$. However similar instabilities occur in various non-supersymmetric D-brane systems. A typical example is a D-brane anti D-brane system. If the distance between the branes is larger than the string scale, an instanton process bridging the two branes can again lead to a runaway hole eating the branes [14]. When the branes are closer than the string scale the same process can take place by a perturbative mechanism. At a critical point the lightest string connecting the branes becomes massless and then tachyonic [15, 16]. This pattern is seen in several examples and leads to the following conjecture:

When the two ends of the world are closer than the 27 dimensional Planck length a tachyon appears in the spectrum. This is just the closed string tachyon found in string perturbation theory.

The action for the instanton (2.6) is proportional to $(r_0/l_p)^{25}$. So in the limit of strong string coupling, $r_0 \rightarrow \infty$, this nonperturbative instability is suppressed. Uncompactified 27 dimensional flat space may be a stable ground state of bosonic M theory.

2.3 Branes

In the absence of supersymmetry, there are no BPS states. Nevertheless, there are stable brane configurations. In terms of the low energy action (2.1) they arise as the extremal limit of black brane solutions. Since the charge must be carried by a four-form, there are 2-branes which are electrically charged and 21-branes which are magnetically charged. It is natural to assume that there are fundamental 2-branes and 21-branes with Planck tension, and these black brane solutions describe the gravitational field of a stack of parallel branes. A nontrivial check of this idea is to compute the tension of a fundamental 2-brane stretched across the extra dimension. It is given by $T = e^\sigma/l_p^2$. Using (2.5), and the relation between σ and ϕ we get $T = 1/l_s^2$ which is the right answer for a fundamental string. Similarly, the tension of a 21-brane which is not oriented along the extra dimension is $T_{21} = 1/l_p^{22} = 1/(g^2 l_s^{22})$ which again is the right answer for a solitonic 21-brane in string theory.

The black brane solutions can be read off from the general discussion of nondilatonic black branes in [17]. The black 2-brane is given by

$$ds^2 = - \left[1 - \left(\frac{r_+}{r} \right)^{22} \right] \left[1 - \left(\frac{r_-}{r} \right)^{22} \right]^{-1/3} dt^2 + \left[1 - \left(\frac{r_-}{r} \right)^{22} \right]^{2/3} dx_i dx^i \\ + \left[1 - \left(\frac{r_+}{r} \right)^{22} \right]^{-1} \left[1 - \left(\frac{r_-}{r} \right)^{22} \right]^{-1} dr^2 + r^2 d\Omega_{23} \quad (2.7)$$

with four-form

$$*F = N l_p^{22} \epsilon_{23} \quad (2.8)$$

where ϵ_{23} is the volume form on a unit S^{23} . The charge N is the number of fundamental two branes, and is related to the two free parameters r_\pm via

$$N^2 = \frac{1100}{3} \left(\frac{r_+ r_-}{l_p^2} \right)^{22} \quad (2.9)$$

There is an event horizon at $r = r_+$ and a curvature singularity at $r = r_-$. The Hawking temperature of this black 2-brane is

$$T = \frac{11}{2\pi r_+} \left[1 - \left(\frac{r_-}{r_+} \right)^{22} \right]^{1/3} \quad (2.10)$$

If we compactify the two directions along the brane on a torus with side L , then the horizon area is

$$A = r_+^{23} \Omega_{23} L^2 \left[1 - \left(\frac{r_-}{r_+} \right)^{22} \right]^{2/3} \quad (2.11)$$

where Ω_{23} is the area of a unit S^{23} . In the extremal limit, $r_+ = r_-$, (2.7) takes a simpler form by setting $\rho^{22} = r^{22} - r_-^{22}$:

$$ds^2 = f(\rho)^{-2/3}[-dt^2 + dx_i dx^i] + f(\rho)^{1/11}[d\rho^2 + \rho^2 d\Omega_{23}] \quad (2.12)$$

where

$$f(\rho) = 1 + \left(\frac{r_-}{\rho}\right)^{22} \quad (2.13)$$

This extremal brane has zero Hawking temperature and is quantum mechanically stable. The surface $\rho = 0$ is a smooth horizon. There is no force between two parallel extremal branes. Static, multi-brane solutions can be obtained by replacing f with a more general solution of Laplace's equation.

We now turn to the black 21-brane. The metric is

$$ds^2 = - \left[1 - \left(\frac{r_+}{r}\right)^3\right] \left[1 - \left(\frac{r_-}{r}\right)^3\right]^{-10/11} dt^2 + \left[1 - \left(\frac{r_-}{r}\right)^3\right]^{1/11} dx_i dx^i \\ + \left[1 - \left(\frac{r_+}{r}\right)^3\right]^{-1} \left[1 - \left(\frac{r_-}{r}\right)^3\right]^{-1} dr^2 + r^2 d\Omega_4 \quad (2.14)$$

and the four-form is $F = N l_p^3 \epsilon_4$, where

$$N^2 = \frac{75}{11} \left(\frac{r_+ r_-}{l_p^2}\right)^3 \quad (2.15)$$

The Hawking temperature is

$$T = \frac{3}{4\pi r_+} \left[1 - \left(\frac{r_-}{r_+}\right)^3\right]^{1/22} \quad (2.16)$$

and the horizon area is

$$A = r_+^4 \Omega_4 L^{21} \left[1 - \left(\frac{r_-}{r_+}\right)^3\right]^{21/22} \quad (2.17)$$

where we have again compactified the directions along the brane to have size L . Setting $\rho^3 = r^3 - r_-^3$, the extremal limit is

$$ds^2 = f(\rho)^{-1/11}[-dt^2 + dx_i dx^i] + f(\rho)^{2/3}[d\rho^2 + \rho^2 d\Omega_4] \quad (2.18)$$

where now

$$f(\rho) = 1 + \left(\frac{r_-}{\rho}\right)^3 \quad (2.19)$$

Like the 5-brane of M theory, this 21-brane is completely nonsingular. The spacetime behind the horizon $\rho = 0$ is identical to the spacetime in front.

We now suppose that one direction of spacetime is compactified on S^1/Z_2 , and the four-form F is odd under the Z_2 identification. The situation is similar to the usual heterotic string construction [18]. Recall that, if y is the coordinate in the compact direction, the components F_{yijk} must be even under $y \rightarrow -y$, while F_{ijkl} are odd (where i, j, k, l denote all directions other than y). Thus, if y is one of the two directions along the 2-brane, the identification can be done trivially since the solution is invariant. As we have already noted, this corresponds to N bosonic strings in 26 dimensions. If the 2-brane is perpendicular to y , a static solution can still be constructed by putting the 2-brane half way between the two fixed points and adding an anti-2-brane at its image point under the Z_2 . (This solution is not known explicitly and will be unstable.) It results in an unstable D2-brane in 26 dimensions. If the 21-brane is perpendicular to y , an invariant solution is obtained by adding another 21-brane (not anti-brane) at its image point under the Z_2 . This corresponds to a 21-brane in string theory which is magnetically charged with respect to the three-form H . If y is one of the directions along the 21-brane, then no invariant solution can be constructed, since $F_{ijkl} \neq 0$ at $y = 0$.

As an aside, we note that there is also a brane solution of 26 dimensional bosonic string theory which has both electric and magnetic charge associated with the three-form H . It is a 21-brane with fundamental strings lying in it and smeared over the remaining 20 directions. Dimensionally reducing to six dimensions by compactifying on a small T^{20} , one recovers the usual self dual black string in six dimensions.

3 Holographic Duals

In this section we go beyond the low energy limit, and try to say something about exact bosonic M theory. Since it contains gravity it should be holographic. There are two types of holographic duals that we have become familiar with. The first is Matrix theory which is based on the existence of stable D0-branes in type IIA theory and the existence of a DLCQ quantization of M theory. However in the present case in which the compactification is on a line interval rather than a circle this type of construction is questionable (but see [8]).

The other type of holographic dual is through AdS/CFT duality [19]. Following the arguments used for the superstring, we consider the near horizon limit of the extreme black brane solutions. As usual, the near horizon limit corresponds to dropping the one in f

in the solutions (2.12,2.18). Starting with the 2-brane, the resulting space is $AdS_4 \times S^{23}$. From (2.9), the radius of each is proportional to $N^{1/11}$. The CFT dual would be a 2+1 dimensional conformal field theory with a global $SO(24)$ symmetry. The natural candidate would be the dimensional reduction of 26 dimensional Yang Mills theory which has 23 scalars in the adjoint representation. This theory has manifest $SO(23)$ symmetry. The mechanism for enhancing the symmetry would have to be similar to the enhancement of $SO(7)$ to $SO(8)$ in the supersymmetric case. However in the present situation we have no superconformal symmetry to ensure the enhanced symmetry. A strong test of the existence of bosonic M theory is the existence of a conformal fixed point with $SO(24)$ symmetry at least in the $N \rightarrow \infty$ limit. In other words, if there does not exist a 2+1 CFT with $SO(24)$ global symmetry, bosonic M theory would be disproven.

As in the usual AdS/CFT correspondence, thermodynamics of the CFT should be related to the near extremal 2-brane. From (2.9) – (2.11), the entropy of near extremal 2-branes can be expressed

$$S \propto N^{25/22}(LT)^2 \quad (3.1)$$

This looks like the entropy of a 2 + 1 field theory. The N dependence is analogous to the $N^{3/2}$ which appears in the usual M 2-brane, and can similarly be viewed as a prediction for the density of states of the theory at strong coupling.

Since there are also solutions of the form $AdS_4 \times K$ where K is any 23 dimensional Einstein space, there may also exist holographic duals of the theory with these boundary conditions. They would be 2+1 conformal field theories with less symmetry.

Starting with the extreme 21-brane (2.18), the near horizon limit is $AdS_{23} \times S^4$, where the radii of each is proportional to $N^{2/3}$. If the theory exists, its holographic dual will be a 22 dimensional conformal field theory with a global $SO(5)$ symmetry. It follows from (2.15)–(2.17) that in the near extremal limit, the entropy of the black 21-brane can be expressed

$$S \propto N^{25/3}(LT)^{21} \quad (3.2)$$

Once again, this is consistent with a 22 dimensional field theory with a large number of degrees of freedom.

4 Discussion

We have proposed that a bosonic version of M theory exists, which is a 27 dimensional theory with 2-branes and 21-branes. One recovers the usual bosonic string by compactifying

on S^1/Z_2 and shrinking its size to zero. In particular, a Planck tension 2-brane stretched along the compact direction has the right tension to be a fundamental string. This picture offers a plausible explanation of the tachyon instability and suggests that uncompactified 27 dimensional flat space may be stable. A definite prediction of this theory is the existence of a $2+1$ CFT with $SO(24)$ global symmetry, which should be its holographic dual for $AdS_4 \times S^{23}$ boundary conditions.

The conjecture that bosonic M theory exists raises a number of questions which we now address:

1) What kind of theory do we get if we compactify bosonic M theory on a circle instead of a line interval? Do we get a weakly coupled string theory in the limit that the circle shrinks to zero? This seems problematic since, whatever the resulting theory is, it should have a massless vector and three-form potential. Of course the open string has a massless vector, but as far as we know, there is no 26 dimensional bosonic string theory with a three-form potential. Instead we believe the limit of bosonic M theory compactified on a circle as the radius $R \rightarrow 0$ is the same as the limit $R \rightarrow \infty$, i.e., the uncompactified 27 dimensional theory. If we compactify bosonic M theory on $S^1 \times (S^1/Z_2)$, and take the second factor very small, this is a consequence of the usual T-duality of the bosonic string. More generally, it appears to be the only possibility with the right massless spectrum.

2) Must bosonic M theory have a vanishing cosmological constant? If not, what is the sign of the cosmological constant? If it is negative then there should be a 27 dimensional AdS solution. The holographic representation of this theory should be an isolated 26 dimensional conformal field theory. Since it is likely that the cosmological constant would be of order one in Planck units we would not expect classical Einstein gravity to be an accurate description. The best description would be the CFT. If the cosmological constant is positive, how do we make sense out of the theory in de Sitter space? This would be the first example of a de Sitter solution emerging out of string theory.

Even with a cosmological constant Λ , there are solutions of the form $AdS_4 \times S^{23}$. The only difference is that the curvature on the two spaces need not be comparable, and are related to different combinations of the four-form charge N and Λ . If $\Lambda > 0$, there is a particularly interesting special case in which the solution is a sphere cross four dimensional Minkowski spacetime. This may have phenomenological applications. It is worth emphasizing that this solution exists for any (positive) cosmological constant, as long as F can be chosen appropriately. It would certainly be interesting to find a dynamical mechanism which would require F to cancel Λ in this way. (For a recent discussion of a

possible mechanism, see [20].) In any event, we find it intriguing that four dimensional spacetimes arise naturally in this theory.

3) Bosonic string theory contains unstable Dp-branes for all p . What are the analog of these in bosonic M theory? It appears that most of these do not survive the strong coupling limit and do not exist as new degrees of freedom in 27 dimensions. This is not surprising since Type II superstring also has unstable Dp-branes which do not appear to have an analog in M theory. However, some Dp-branes may remain. We already saw a construction of an unstable D2-brane in section 2. D0-branes can be identified with modes in the 27^{th} direction. In the theory compactified on S^1/Z_2 , these modes are unstable since they bounce off the fixed points, interact with themselves and decay into radiation in the other directions. In the uncompactified limit, they should become stable.

4) Even if 27 dimensional flat space, M_{27} , is a stable vacuum, one might ask what is the “ground state” of the theory at finite string coupling, or finite compactification size? Tachyon condensation is not likely to lead back to M_{27} , and there is probably no stable minimum of the tachyon potential in 26 dimensions [7]. Instead, we believe tachyon condensation may lead to an exotic state with zero metric $g_{\mu\nu} = 0$. It is an old idea that quantum gravity may have an essentially topological phase with no metric. We have argued that the tachyon instability is related to nucleation of “bubbles of nothing” which is certainly reminiscent of zero metric. Further support for this idea comes from some old results on the closed string tachyon. Using modular invariance of the one loop vacuum amplitude, one can relate the existence of a tachyon to the asymptotic density of states. It was shown that the tachyon is absent only if, at high energies, the theory has at most a finite number of fields propagating in two spacetime dimensions [21]. Similar results were found by studying the theory near the Hagedorn transition [22]. If the theory starts in 26 (or 27) dimensions, the only way to get down to two dimensions is to have a highly degenerate metric. The most symmetric state would then be $g_{\mu\nu} = 0$, and two dimensional subspaces might arise as excitations.

This raises an interesting question in string field theory. Witten’s open bosonic string field theory [23] takes the form

$$S = \int A * QA + \frac{2g}{3} A * A * A \quad (4.1)$$

where Q is the BRST operator and $*$ is a noncommutative product. Formally, \int and $*$ are independent of the metric and other closed string backgrounds but Q is not. Since an interacting theory of open strings must include closed strings, it is awkward having this

explicit background dependence in the action. It was shown in [24] that (4.1) could be derived from the purely cubic action

$$S = \int \Phi * \Phi * \Phi \quad (4.2)$$

There is a solution Φ_0 to the equation of motion $\Phi * \Phi = 0$ such that expanding about this solution, $\Phi = \Phi_0 + g^{1/3}A$, one recovers Witten's action. The natural ground state of the purely cubic action is $\Phi = 0$. Since this corresponds to zero BRST operator, it has been interpreted as a state of zero metric. But the purely cubic action can be viewed as the strong coupling limit $g \rightarrow \infty$ of (4.1)³. If this is similar to the strong coupling limit of purely closed bosonic string theory, the natural ground state should be M_{27} . Could it be that $\Phi = 0$ really corresponds to M_{27} and the fact that $Q = 0$ is just the statement that there are no open string excitations? If so, how can one recover the metric and three-form excitations?

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